A method to correct for the «brighter-fatter» effect

Augustin Guyonnet - BNL Paccd Meeting (2014)

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PARIS

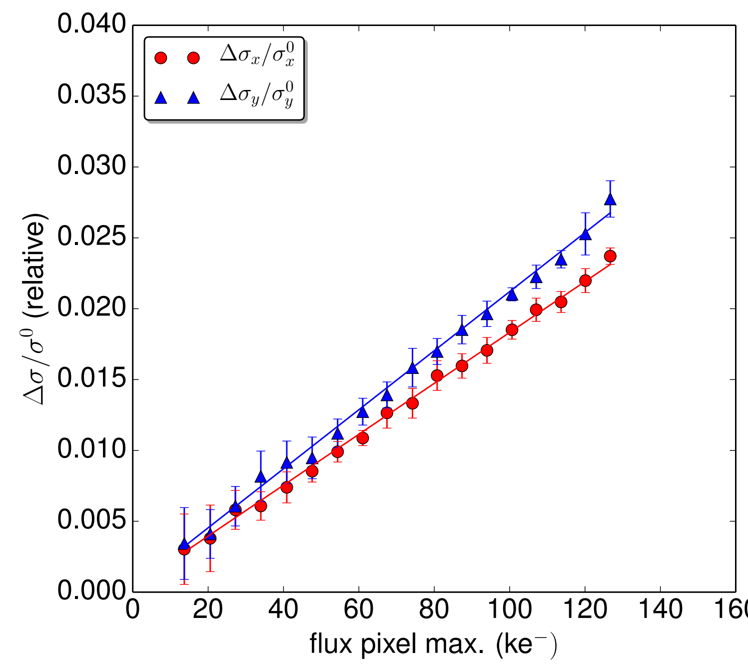
the «brighter-fatter effect»

The brighter-fatter effect is the observation that the width of stellar images or laboratory luminous spots increases with the flux of the object.

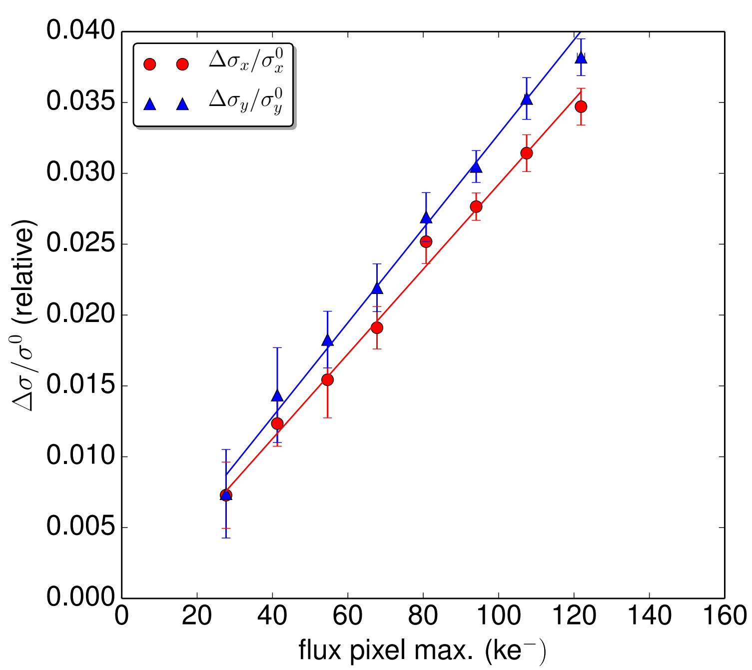
- It has been seen by all the telescopes that looked for it,
- This is linear with the flux,
- This is slightly asymetric,
- There is no chromaticity detected.
- It is caused by the collected charges which change the surrounding electric field.
- It also manifests in flatfield images by spatially correlating the pixels.

the «brighter-fatter effect»

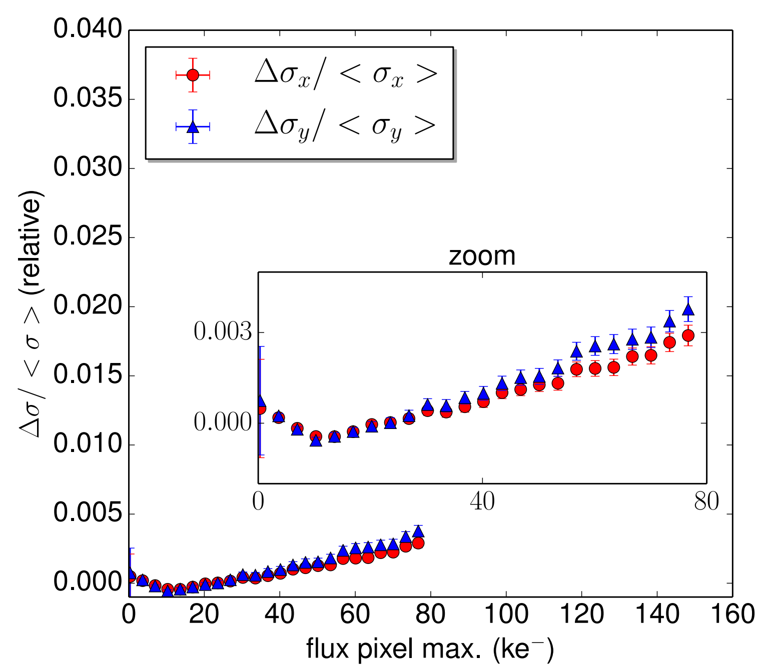
	X		Y	
	σ (pix)	$\Delta\sigma$@100ke$^-$ (pix)	σ (pix)	$\Delta\sigma$@100ke$^-$ (pix)
CCD E2V-250 - 550nm	1.594	0.047 ± 0.002	1.622	0.052 ± 0.003
CCD E2V-250 - 900nm	2.042	0.037 ± 0.0005	2.048	0.043 ± 0.0007
DECam - r-band ($\sim$ 640nm)	1.709	0.022 ± 0.001	1.944	0.024 ± 0.001
MegaCam - r-band ($\sim$ 640nm)	1.980	$0.005 \pm$ ns	1.960	$0.006 \pm$ ns



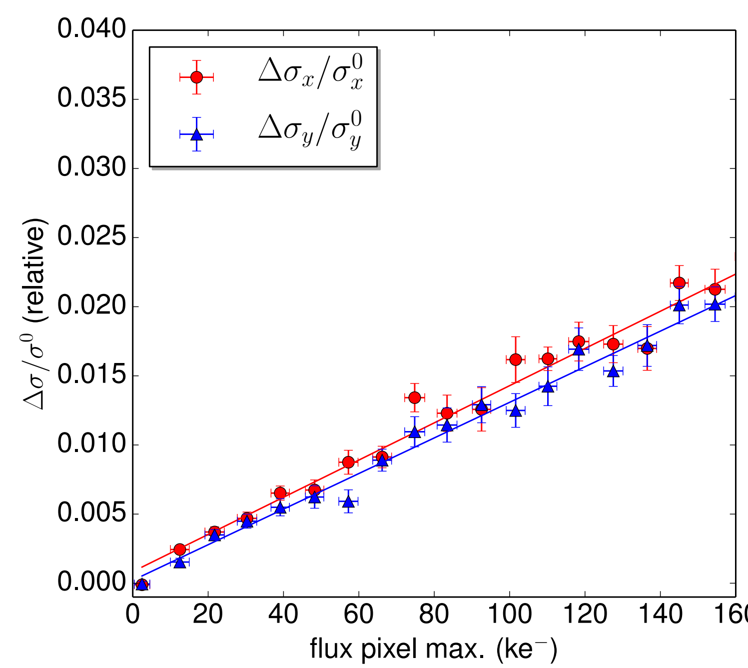
(b) LSST - E2V 250 - Spots 900 nm



(a) LSST - E2V 250 - Spots 550 nm



(c) MegaCam - E2V 42-90 - r-band stars

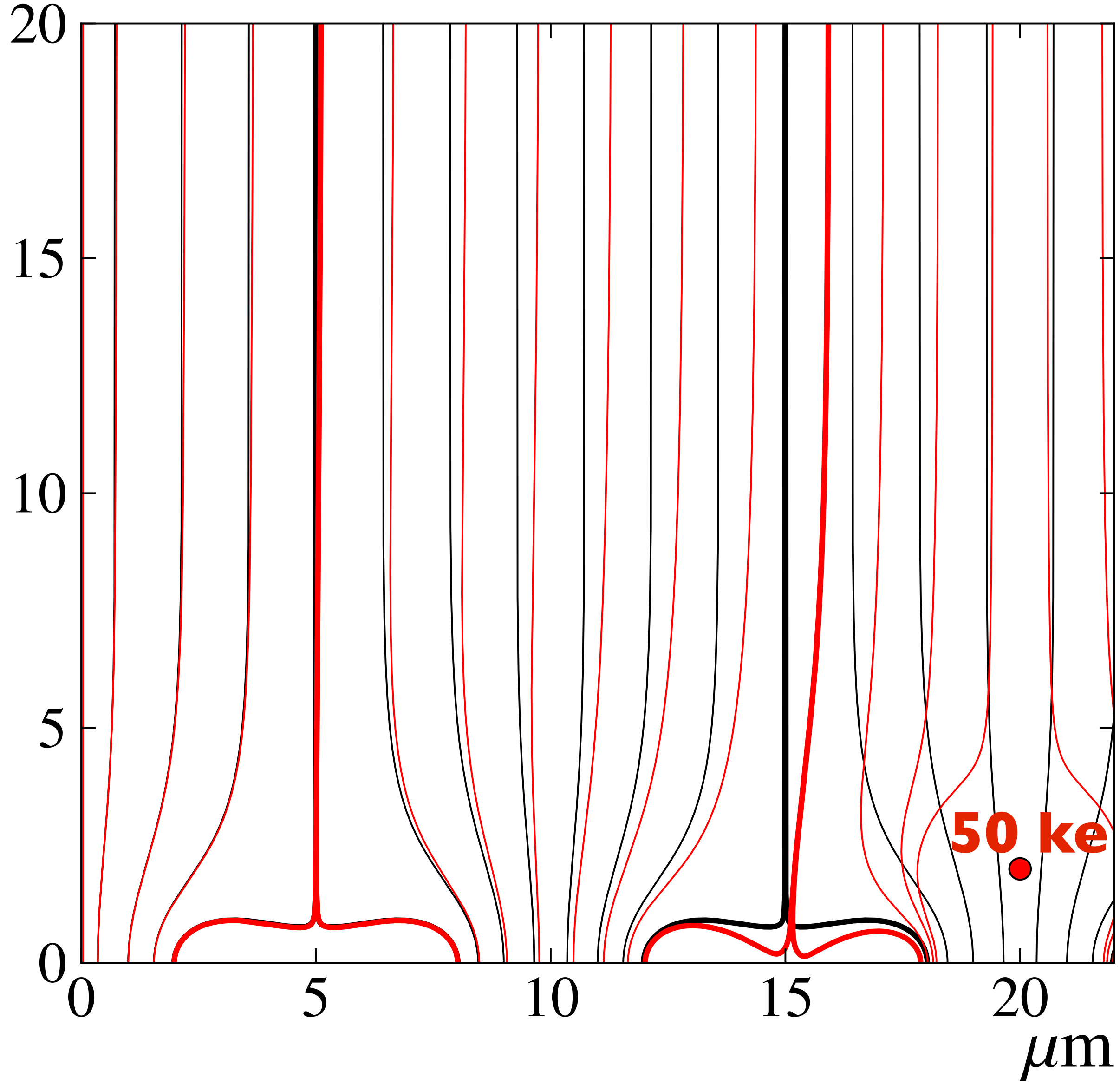


(d) DECam - LBL/DALSA - r-band stars

Physics of the effect

Evolution of the electrostatic field due to collected charges :

- **Field lines displacement**

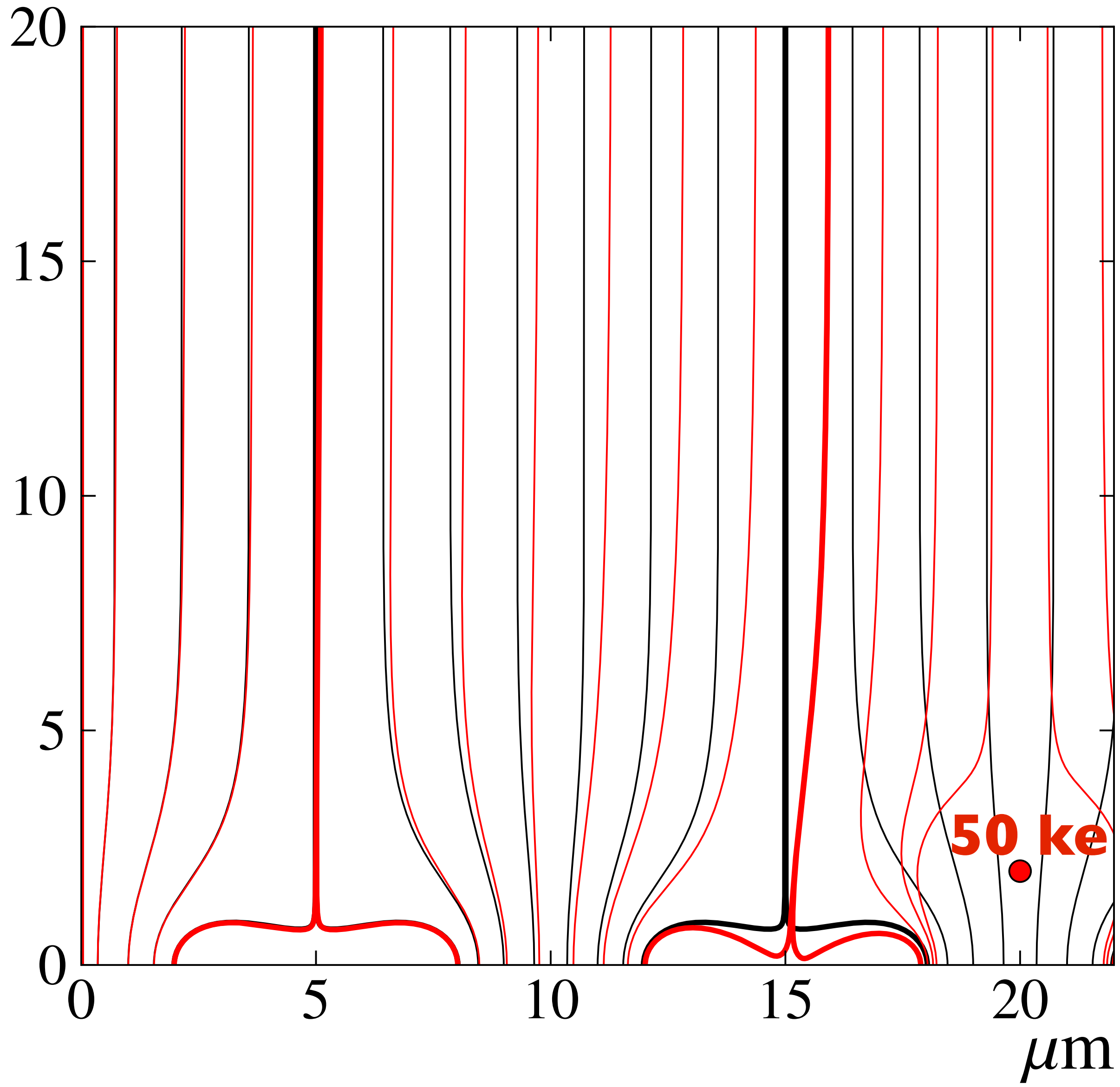


Physics of the effect

Evolution of the electrostatic field due to collected charges :

$$\sigma_{PSF} = \sqrt{2Dt_r}$$

- Field lines displacement



- Electric potential diminution

size (µm)	CCD E2V-250 X	CCD E2V-250 Y	DECam X	DECam Y
Initial PSF	15.94	16.22	25.64	28.86
PSF at 100 ke$^-$	16.41	16.74	25.97	29.18
Observed increase	0.47	0.52	0.33	0.32
Diffusion (σ_{PSF})	<4.00	<4.00	<7.00	<7.00
Diffusion induced increase at 100 ke$^-$	0.018	0.018	0.067	0.067
Diffusion contribution (%)	3.7	3.4	20.2	20.7

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Physics of the effect

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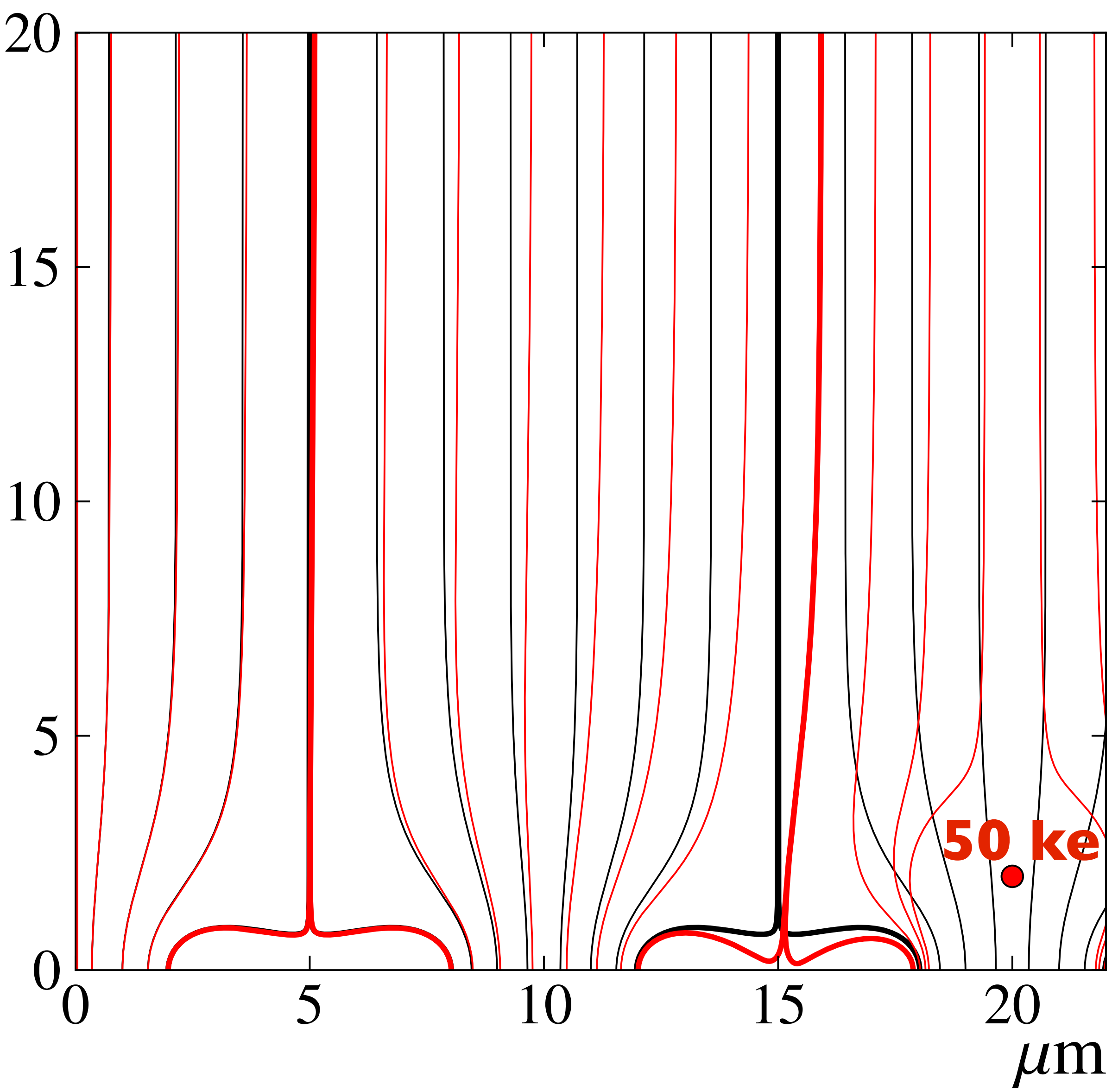

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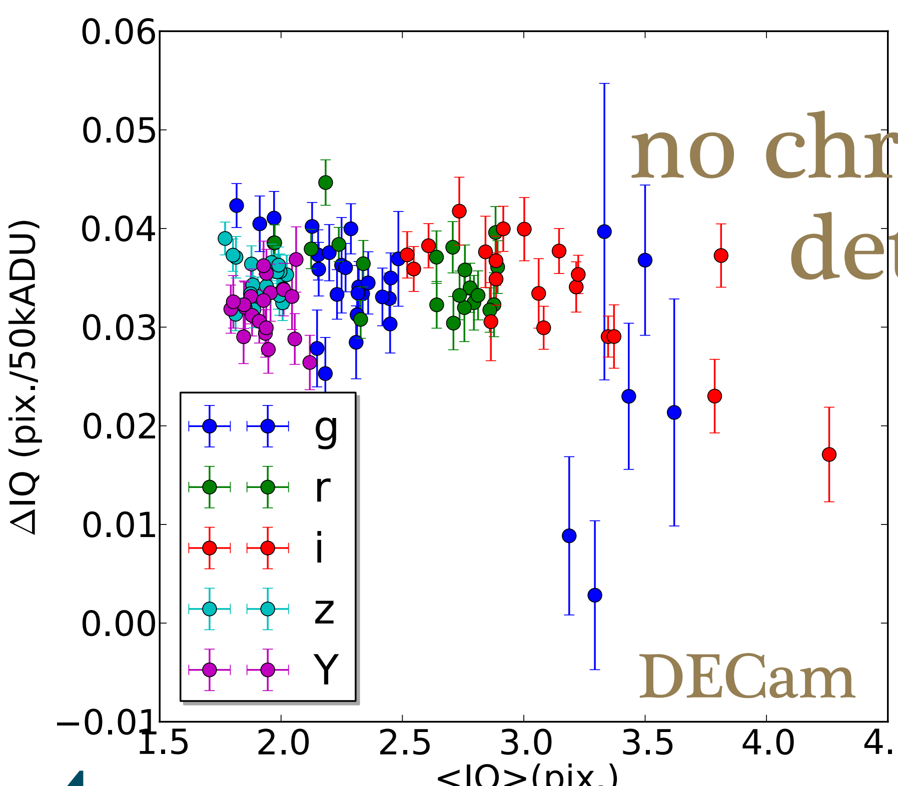

no chromaticity detected

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Impact of the «brighter-fatter» effect on PSF photometry

PSF Photometry is defined by a least square function :

$$\chi^2 = \sum_p (I_p - \Phi\ PSF)\ w_p$$

sum over pixel

sky-subtracted image

flux estimator

pixel weights

Solving for $\dfrac{d\chi^2}{d\Phi} = 0$ gives the

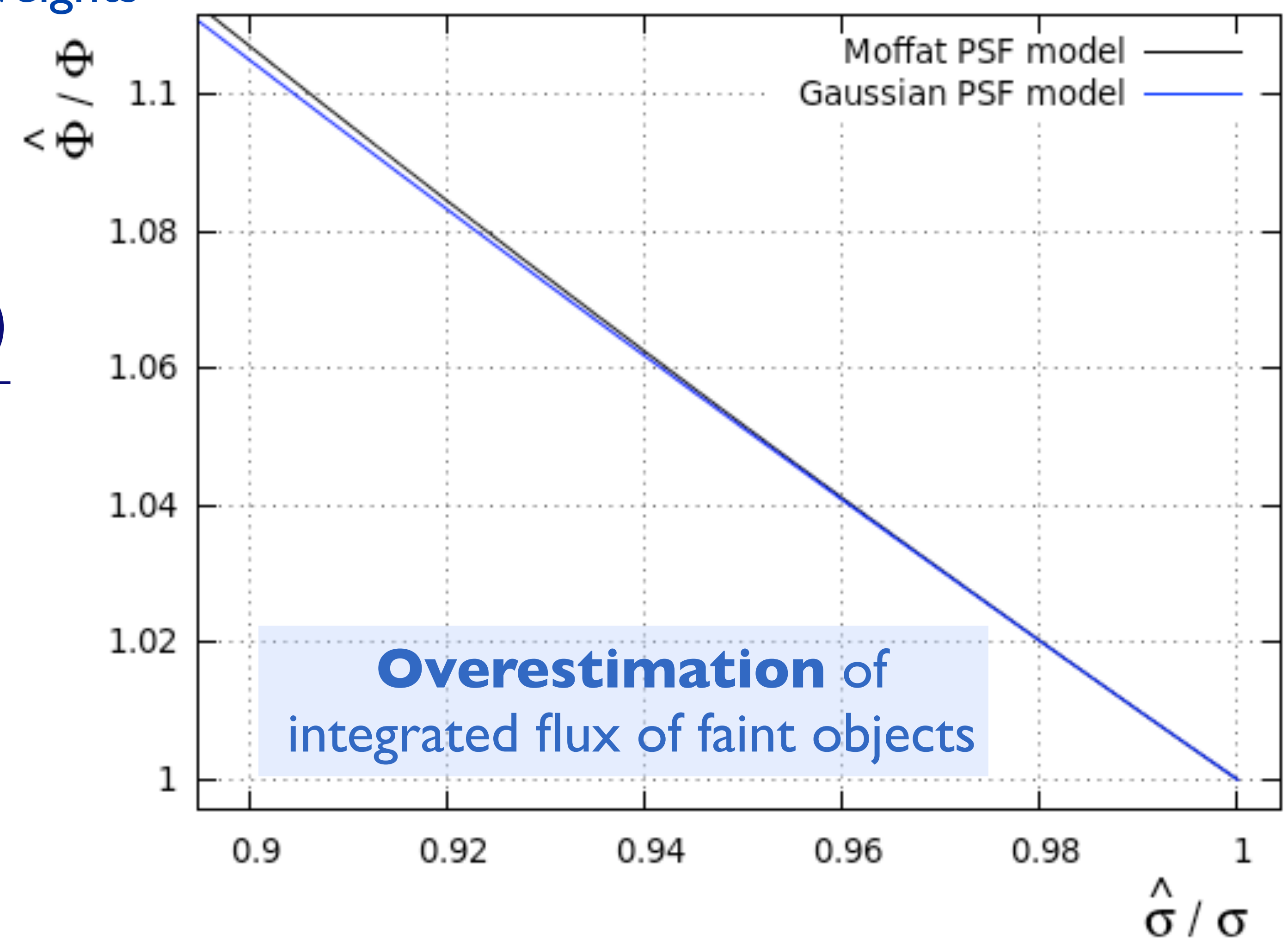
flux estimator : $$\Phi = \frac{\sum_p (w_p I_p\ PSF)}{\sum_p (w_p\ PSF^2)}$$

Assuming that the faint object has an actual PSF smaller than the one of the model:

$$I_p = P\hat{S}F$$

An error in the PSF translates in an error on the flux the following way :

$$\frac{\hat{\Phi}}{\Phi} = \frac{\sum_p (PSF \cdot P\hat{S}F)}{\sum_p PSF^2}$$



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What could we do?

Solutions could be :

=> to change the psf model ?

=> to process the image to correct for the effect prior to psf modelling.

Prediction from electrostatic simulation

build an empirical model and ajust it on the measurement of the effect in flatfield images

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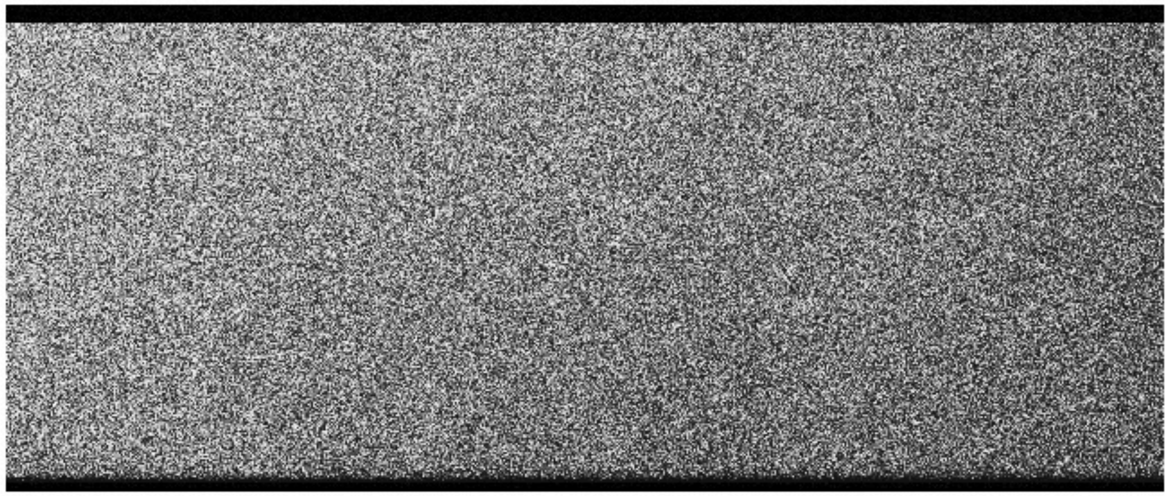
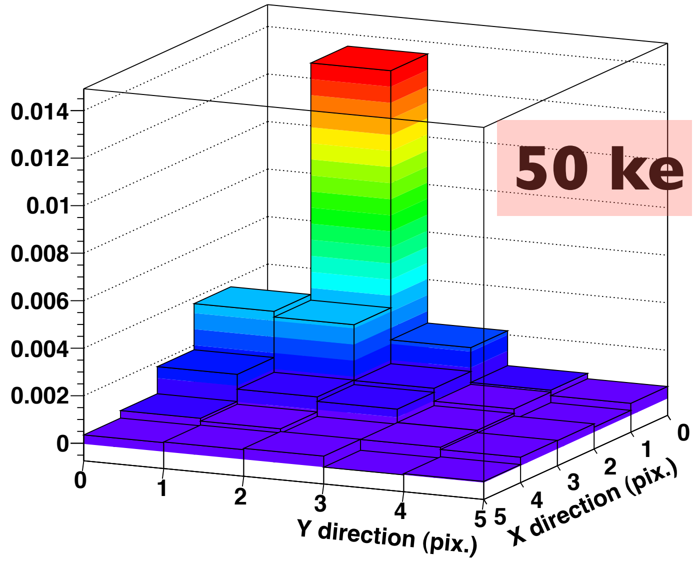
Pixel spatial correlations in flatfield

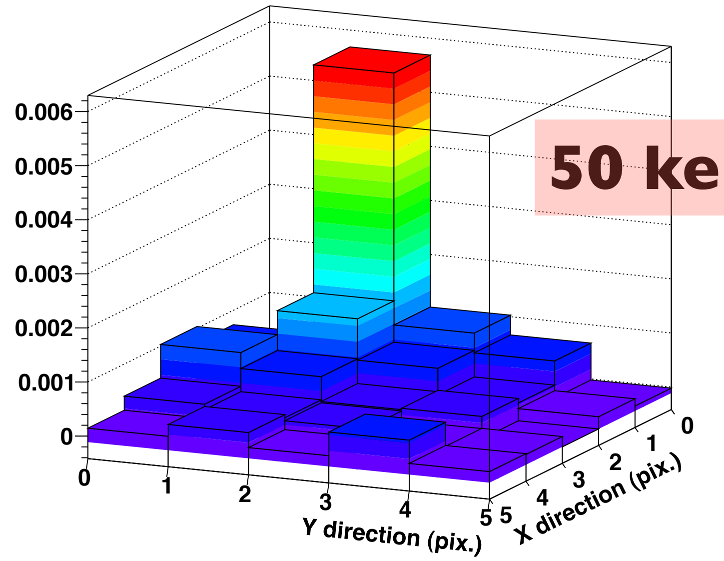


Difference from a pair of flatfields: Pixels with higher counts come from Poissonian fluctuation.

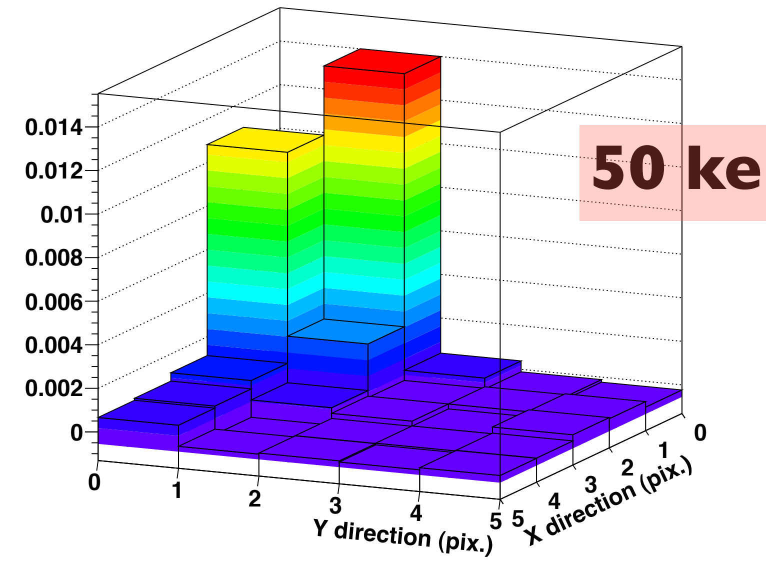
Pixels with higher counts modify the electric field in their surrounding, it spatially correlates pixels.



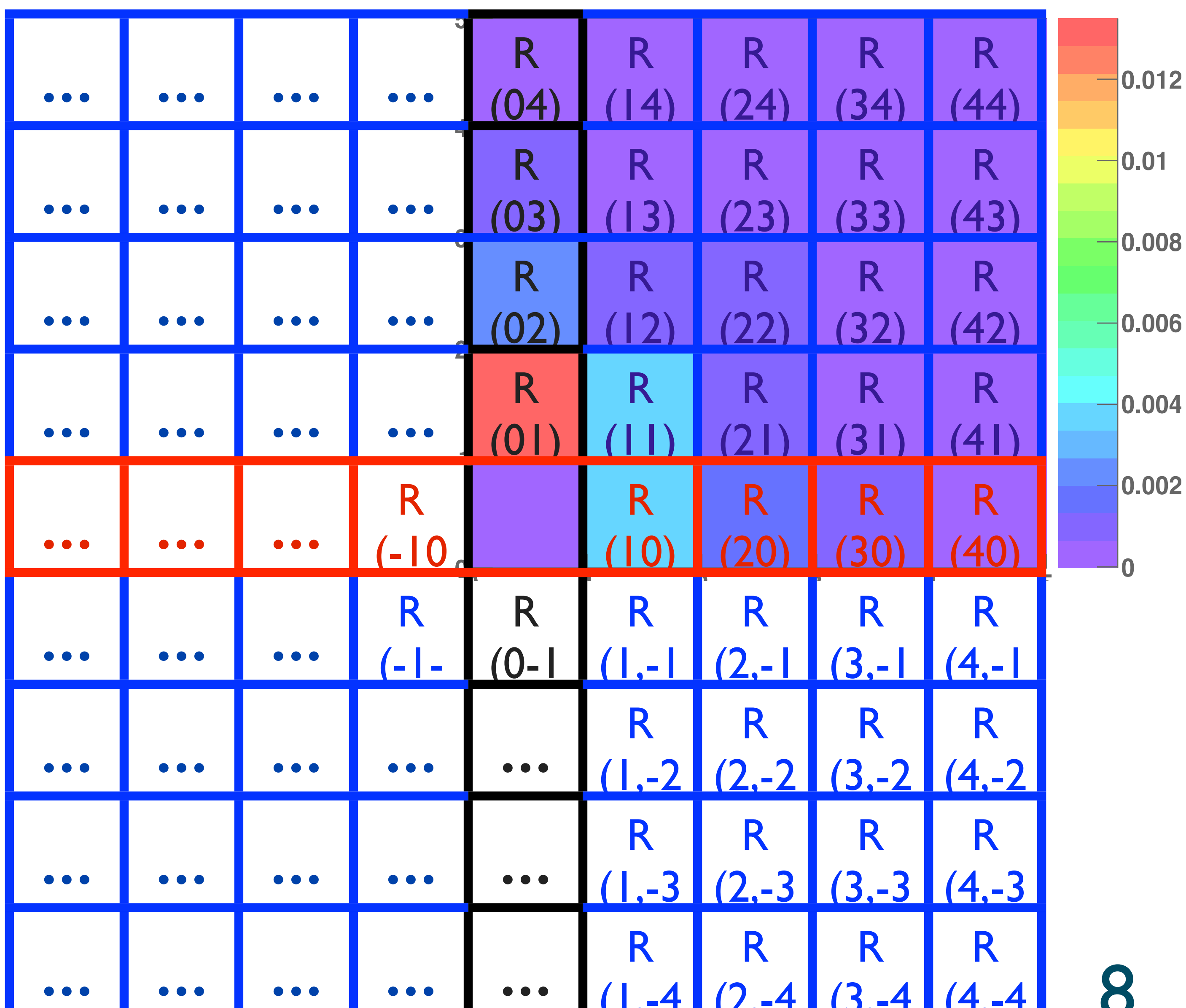
(a) LSST - CCD E2V 250

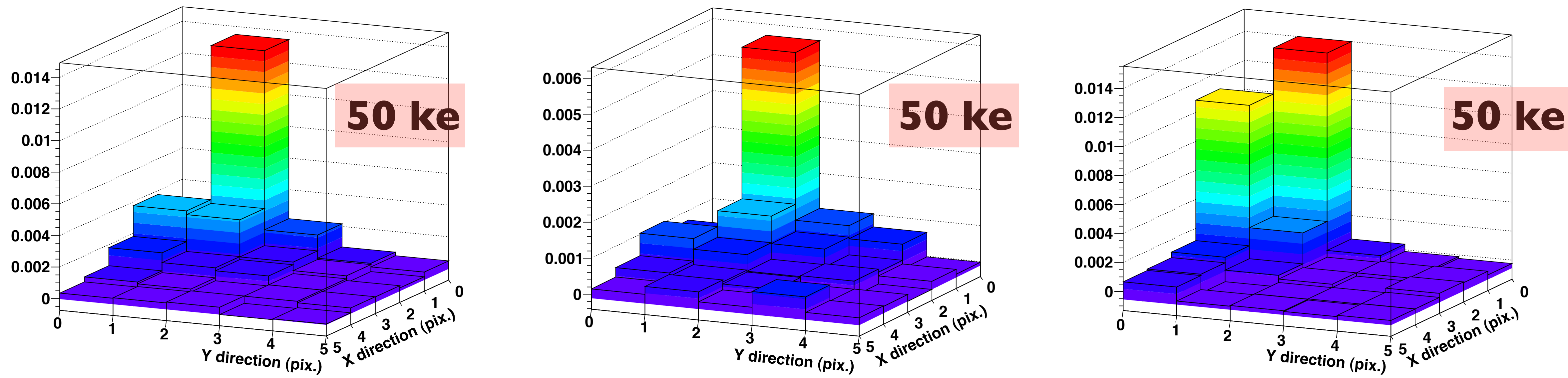


(b) DECam - CCD S11

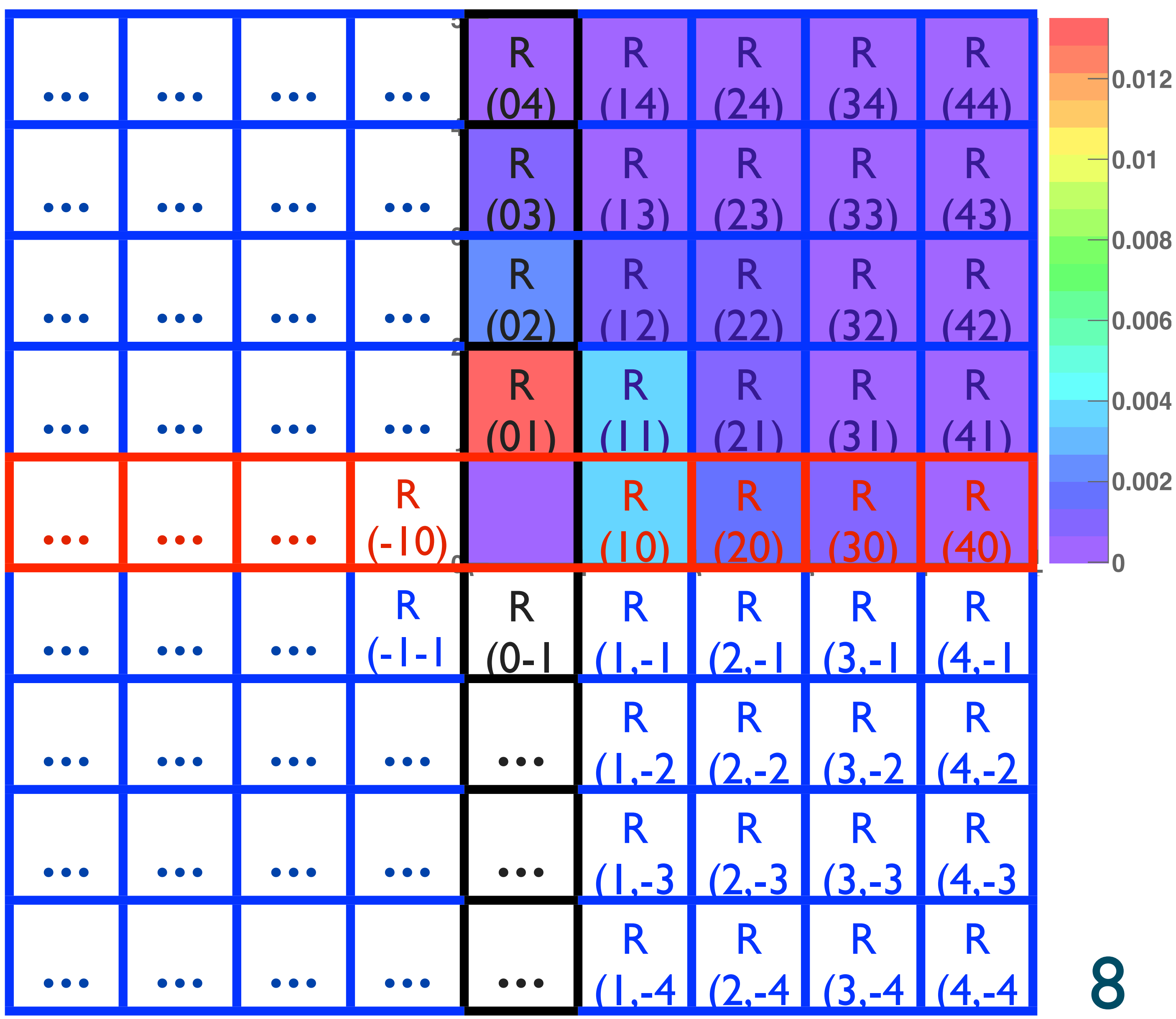


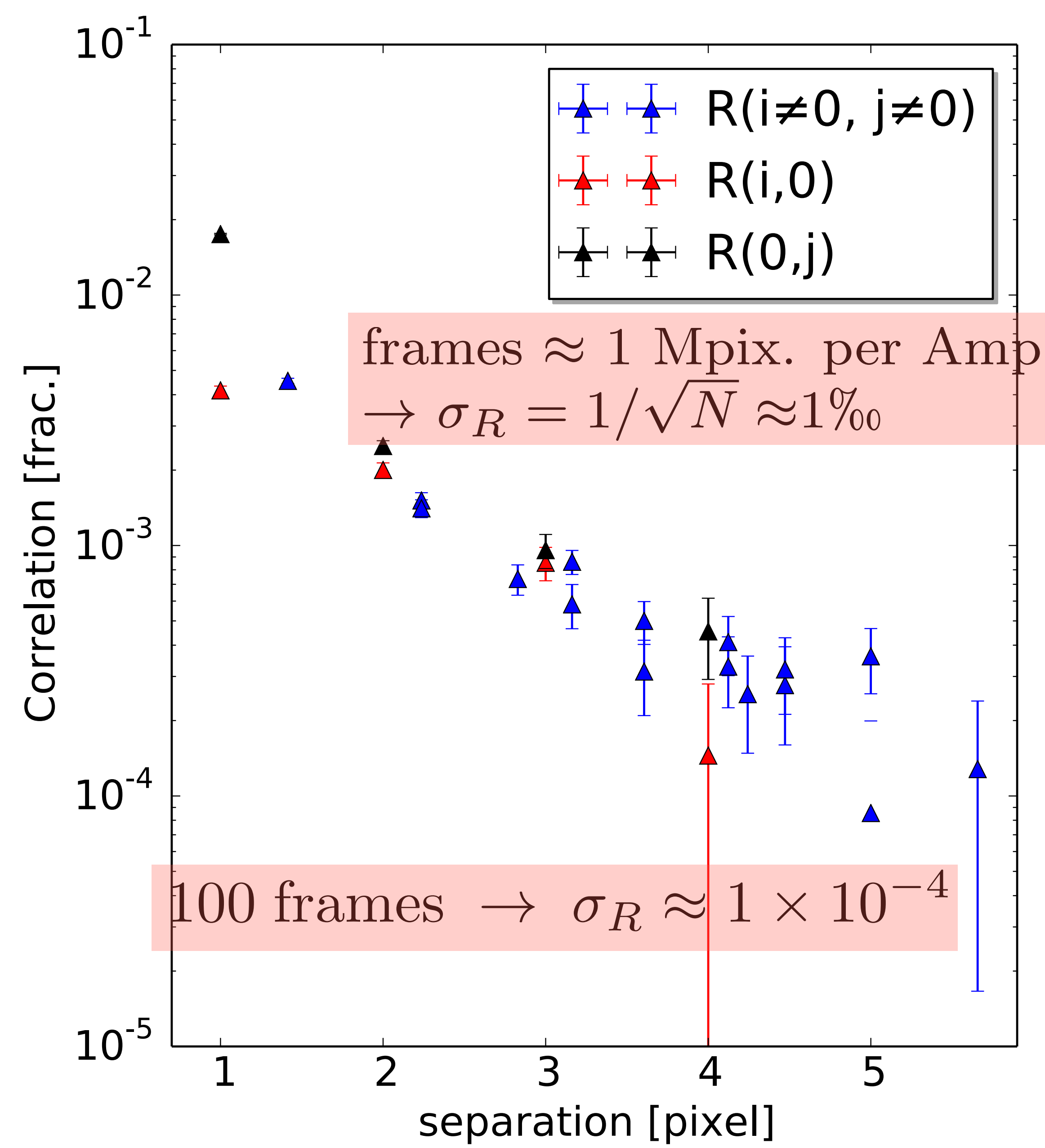
(c) MegaCam





(a) LSST - CCD E2V 250 (b) DECam - CCD S11 (c) MegaCam



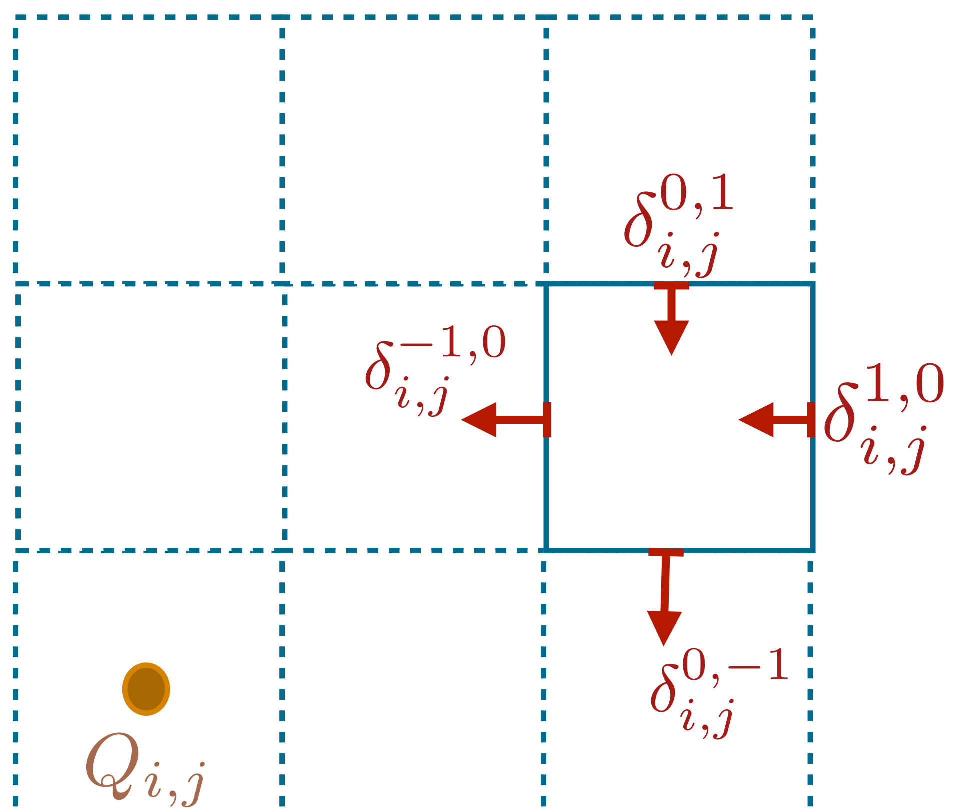


Pixel effective size model - step 1

- Charges stored in a pixel source an electric field ...

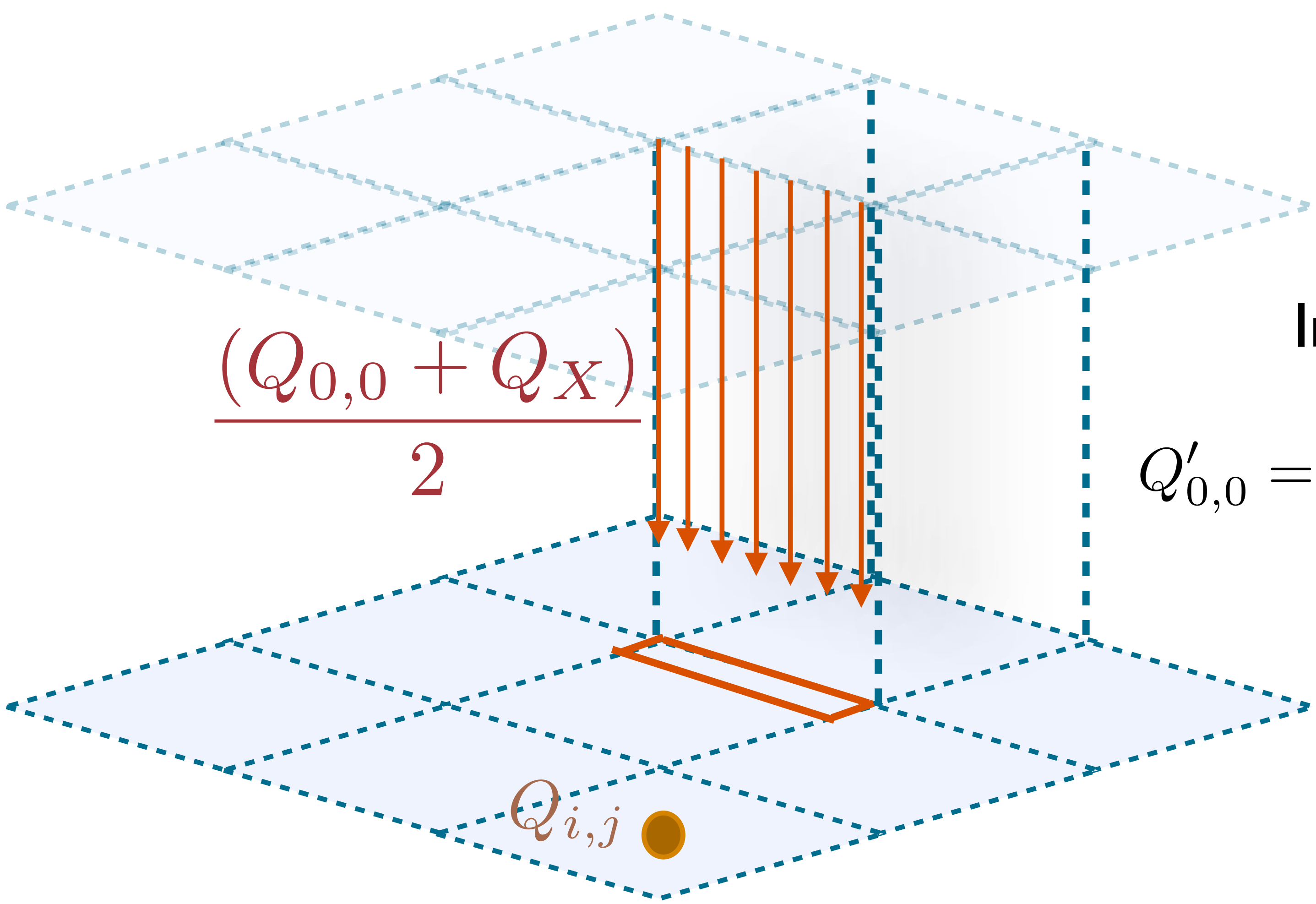
For a pixel in the surrounding, it results in a boundary displacement :

$$\frac{\delta^X}{p} = \frac{1}{2}\sum_{i,j} Q_{i,j} a^X_{i,j}$$



Pixel effective size model - step 2

... which affects incoming charges.



In the perturbed flatfield image :

$$Q'_{0,0} = Q_{0,0} + \underbrace{\sum_X \sum_{i,j} \frac{1}{2} a^X_{i,j} Q_{i,j} \frac{(Q_{0,0}+Q_X)}{2}}_{\text{Charge «transfer»}}$$

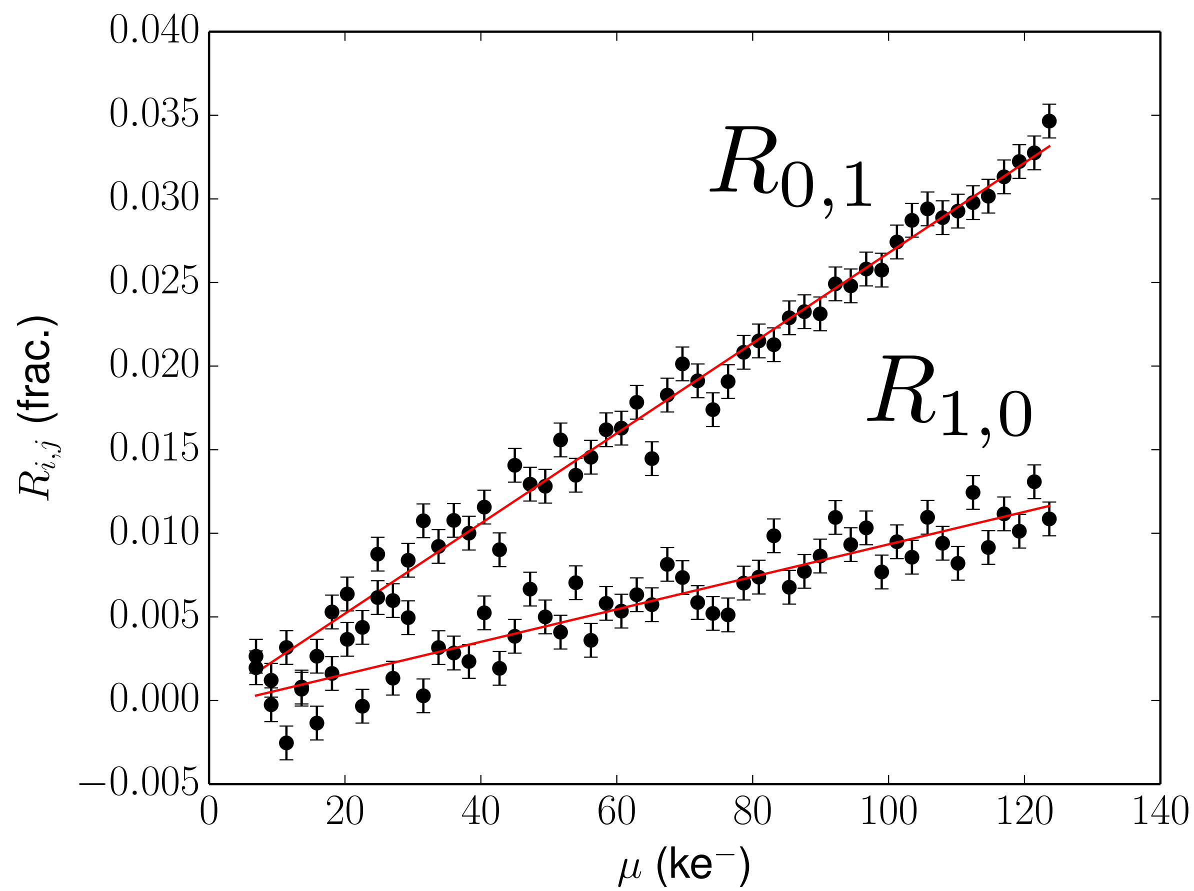
Pixel effective size model - step 3

A little algebra *(Antilogus et al. 2014)* relates the (a) to the covariances :

$$Cov(Q'_{i,j}, Q'_{0,0}) = V\mu \sum_X a^X_{i,j}$$

Or, equivalently,
the slope of a correlation (Rij) :

$$\frac{R_{i,j}}{\mu} = \sum_X a^X_{i,j}$$



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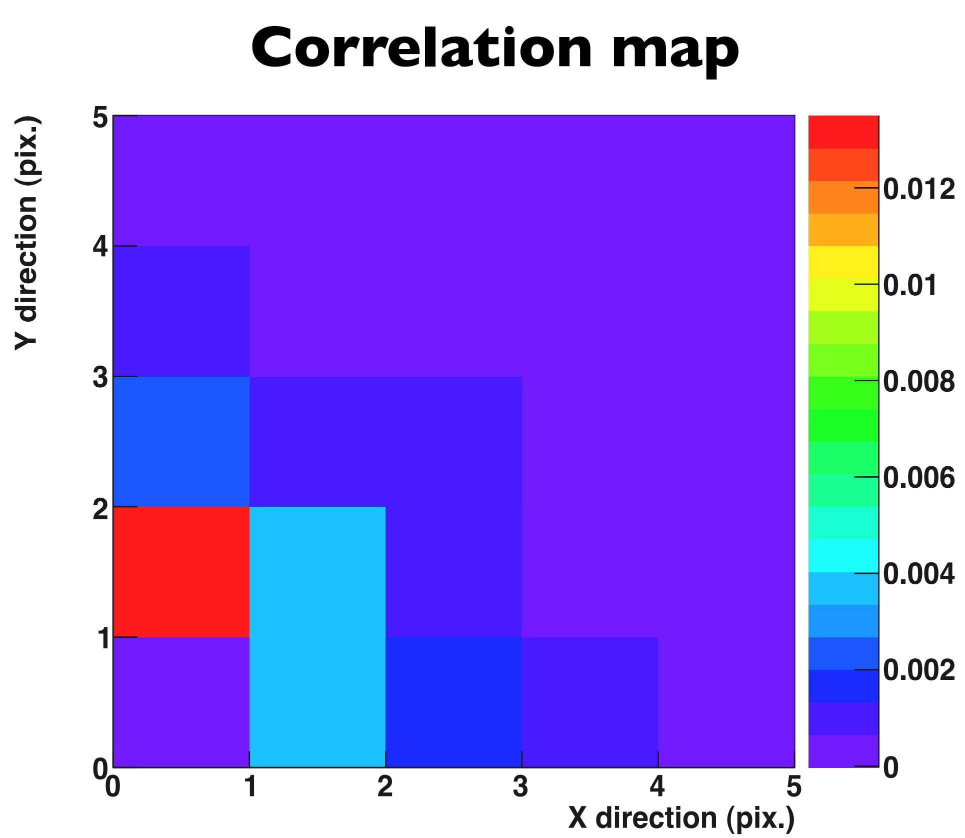
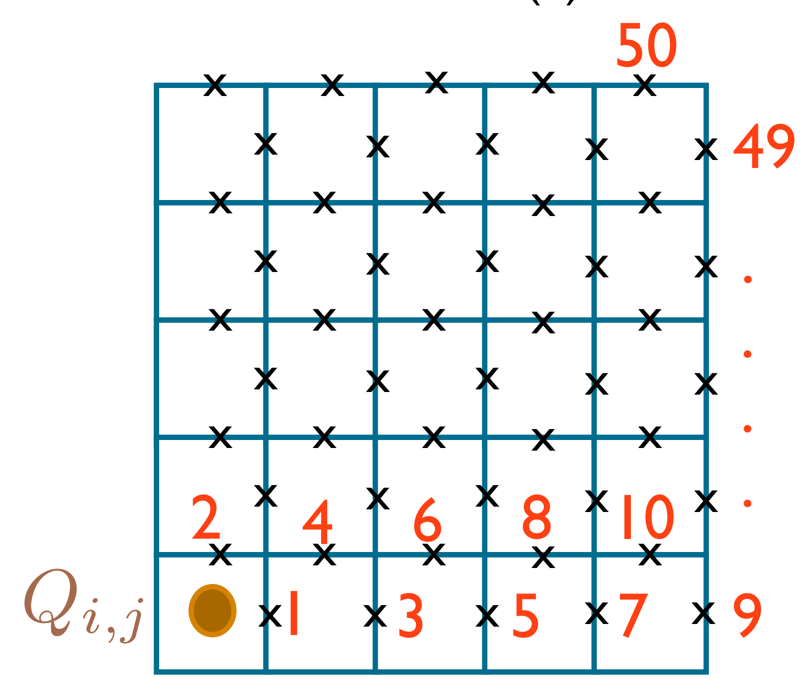
Applying the model to the 5 by 5 correlation map of the CCD E2V-250



There are 24 Ri,j measurements.

For i,j = 0, ..., 4 there are 50 (a) terms to evaluate,

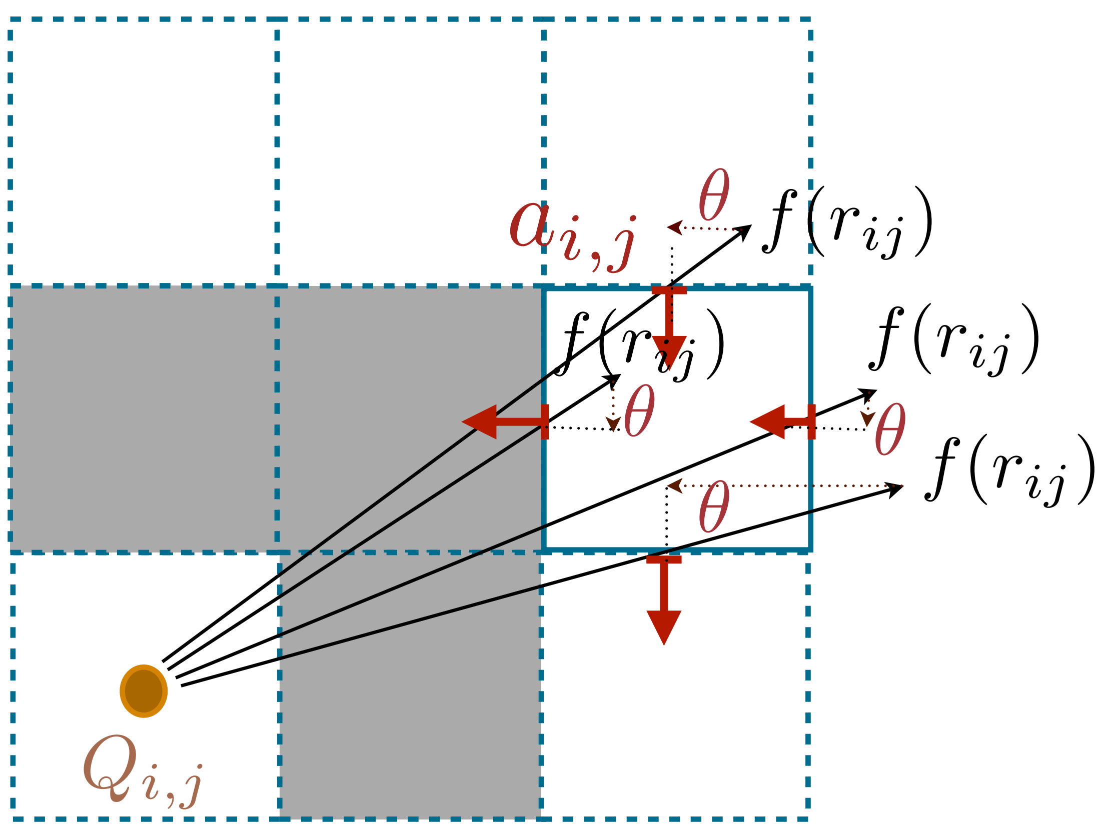


Added constraints to determine the (a) coefficients

For the missing constraints,
we apply a model to the displacement of the non-nearest boundaries.

We suppose that the electrostatic force is isotropic
and that it is a smooth function of the distance.
The (a) coefficients correspond to its projection normal to the boundary :

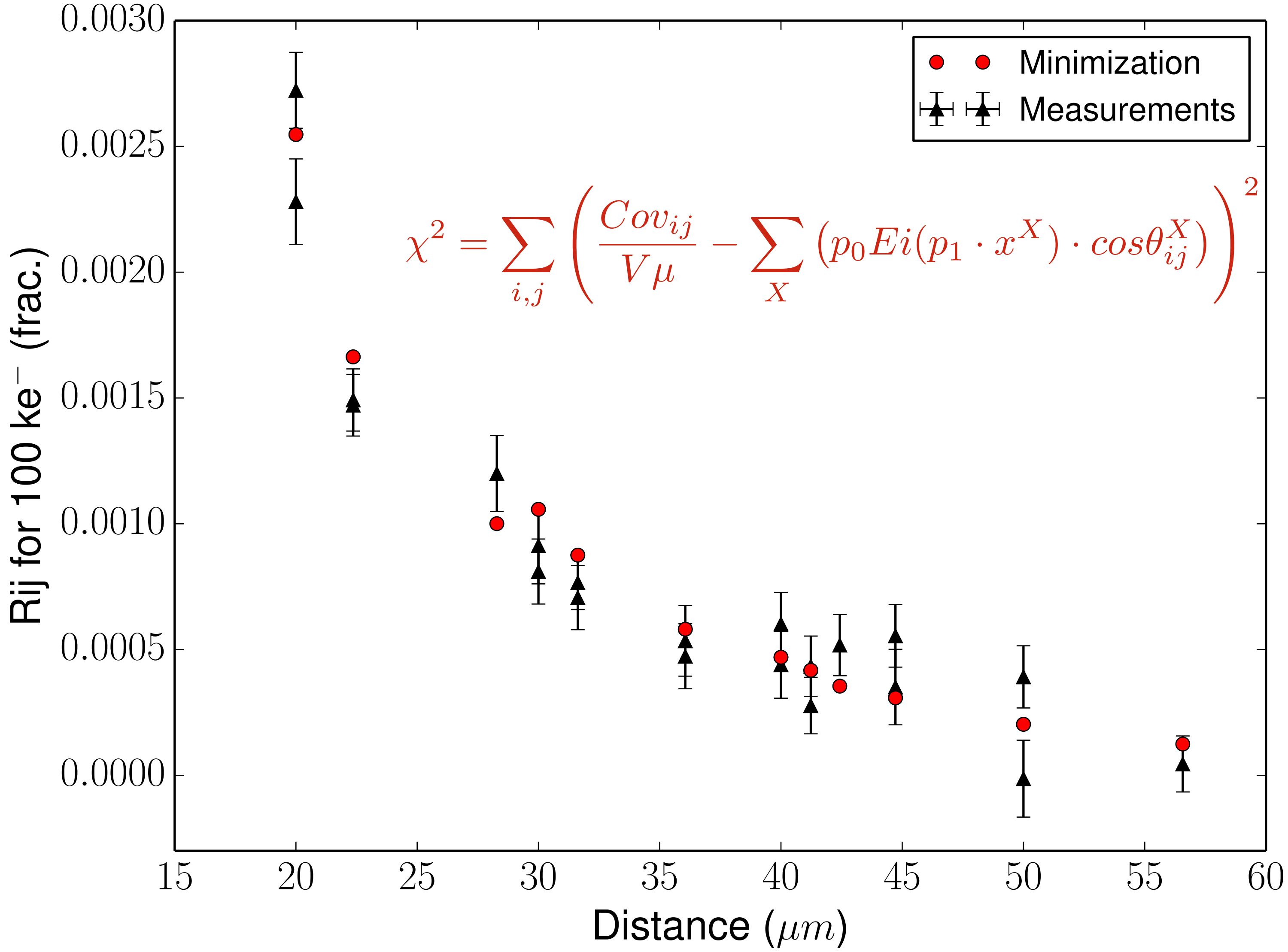
$$a_{i,j}^{X} = f(r_{ij}) \cos \theta_{i,j}^{X}$$



Projection of the electrostatic force for non-nearest boundaries displacement

We directly minimize against the correlation measurements :

$$\chi^2 = \sum_{i,j} \left(\frac{Cov_{ij}}{V\mu} - \sum_X \left(f(r_{ij}^X) \cdot cos\theta_{ij}^X \right) \right)^2$$



And we settle for :

$$f(r) = p_0 Ei(p_1 r)$$

$$Ei(x) \equiv - \int_{-x}^{\infty} \frac{e^{-t}}{t} dt$$

Solution of the model : a system of 50 equations

To solve our initial system of 50 parameters :

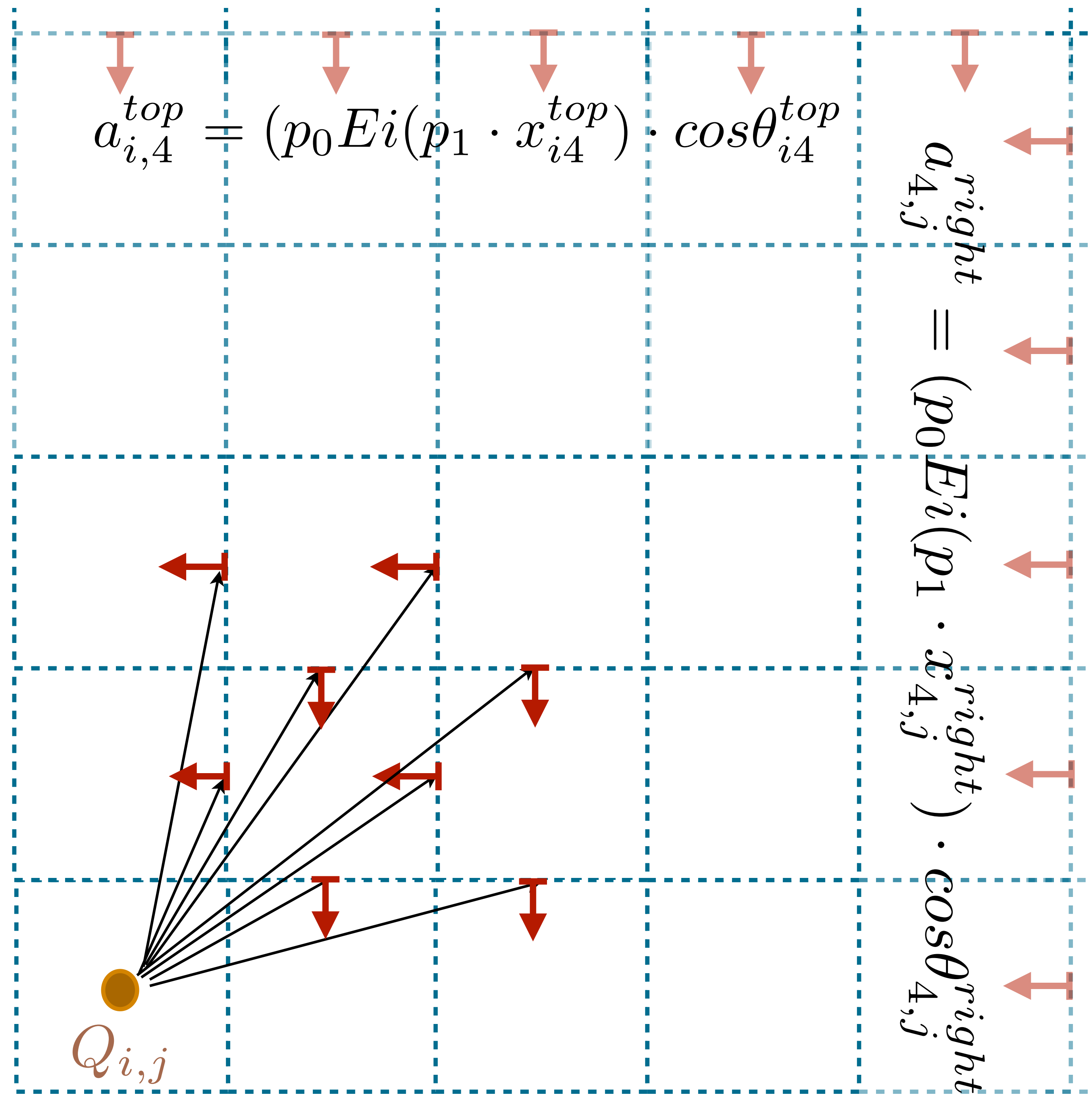
We combine the 24 $\frac{R_{i,j}}{\mu} = \sum_X a_{i,j}^X$

with the determination of10 limit conditions using the isotropic parametrization:

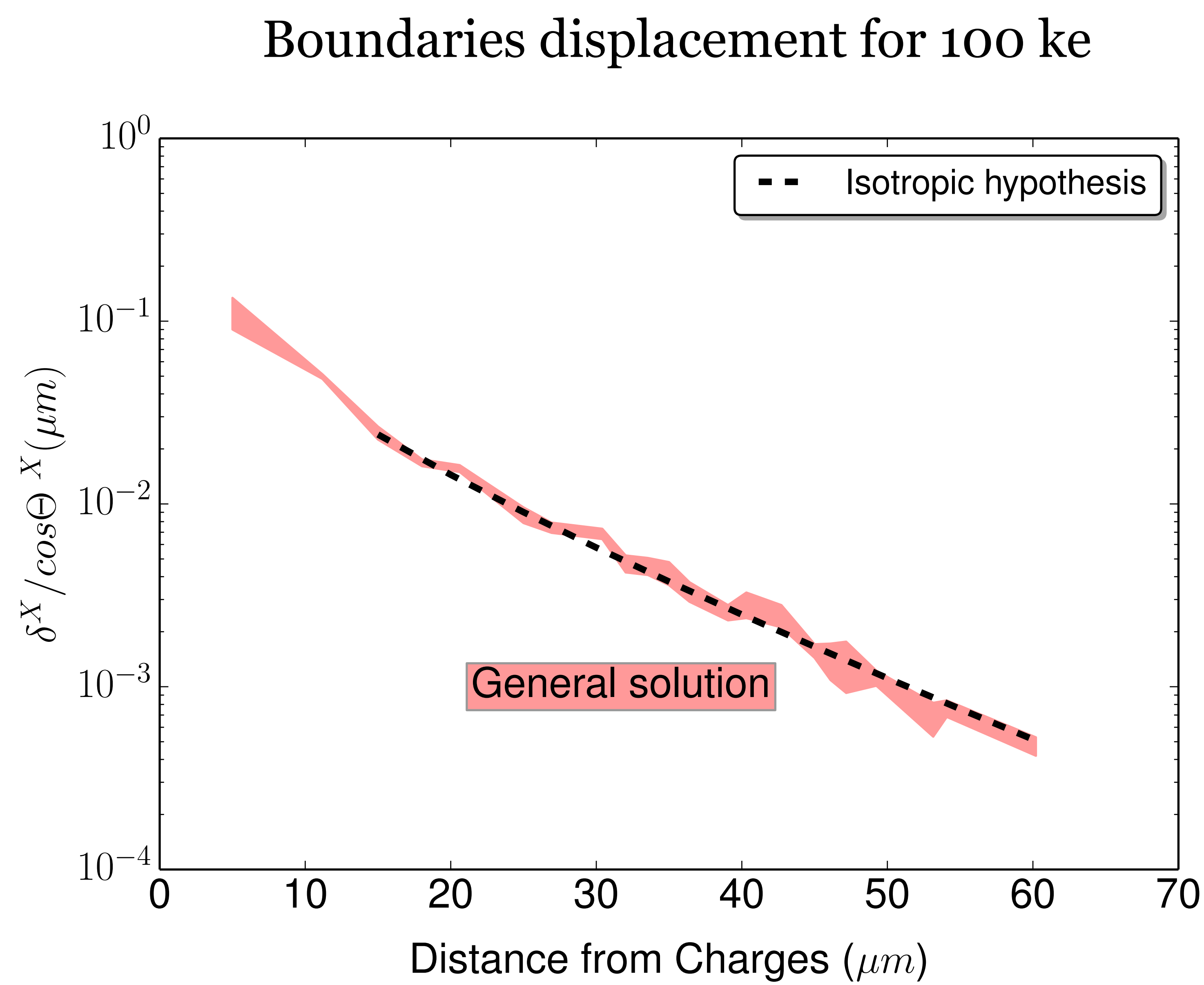
$$f(r) = p_0 Ei(p_1 r)$$

And we also apply the relative variation of the isotropic parametrization to adjacent pairs of the off-axis boundaries:

$$a_{i,j}^{(0,-1)} = a_{i,j}^{(-1,0)} \left(\frac{Ei(p_1 \cdot r_{i,j}^{(-1,0)}) \cdot cos(\theta_{i,j}^{(0,-1)})}{Ei(p_1 \cdot r_{i,j}^{(0,-1)}) \cdot cos(\theta_{i,j}^{(-1,0)})} \right)$$



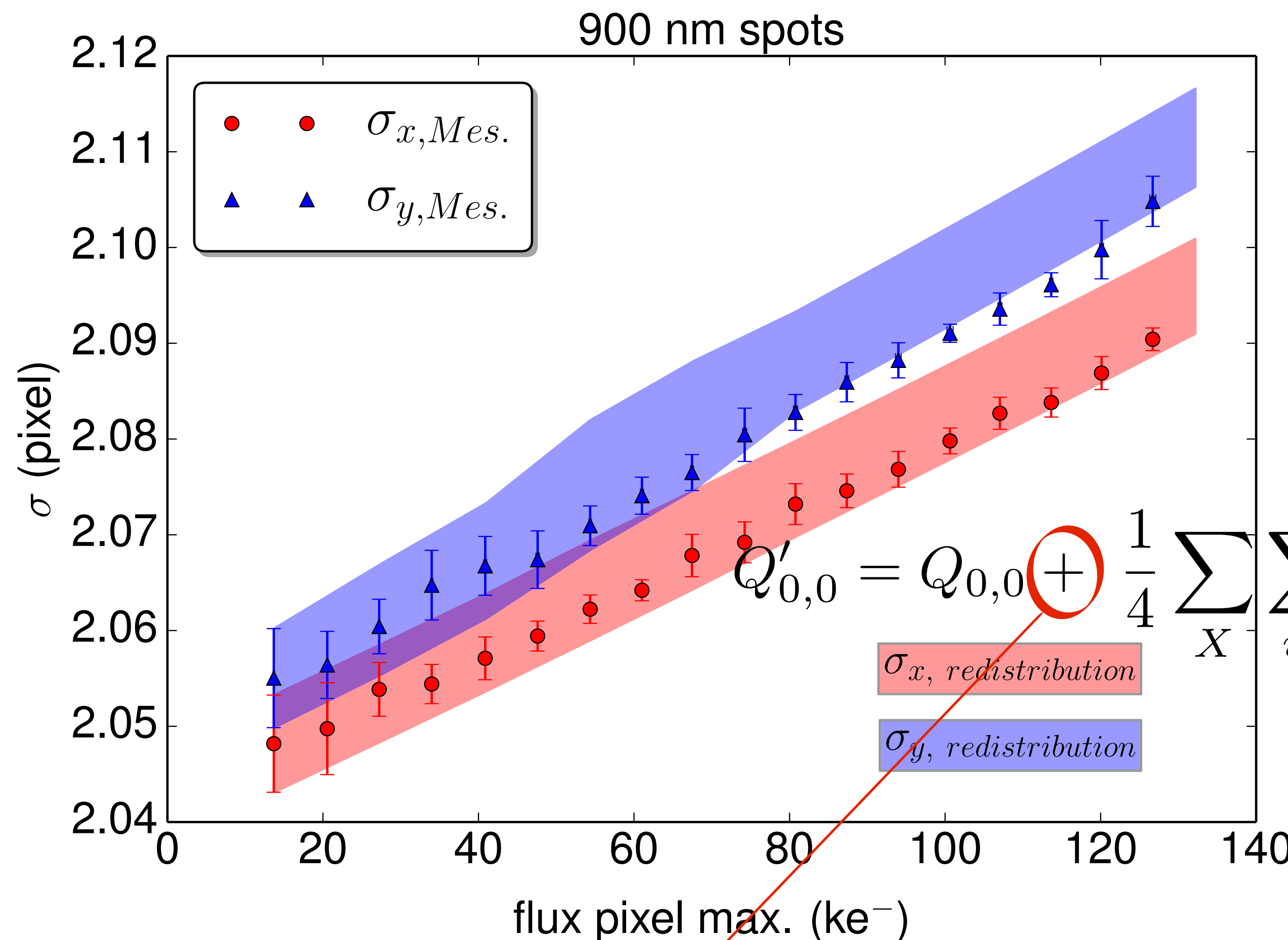
Boundaries displacement



The solution is then replicated to the 3 other quadrants

Comparing the model to the data

10 images of a spot at each exposure time. Fitted with a 2 Gaussian.



$$Q'_{0,0} = Q_{0,0} + \frac{1}{4} \sum_X \sum_{i,j} a^X_{i,j} Q_{i,j} (Q_{0,0} + Q_X)$$

Reverse model as a post-processing method to move flux back to where it belongs

$$Q_{0,0} = Q'_{0,0} - \frac{1}{4} \sum_X \sum_{i,j} a^X_{i,j} Q_{i,j} (Q_{0,0} + Q_X)$$

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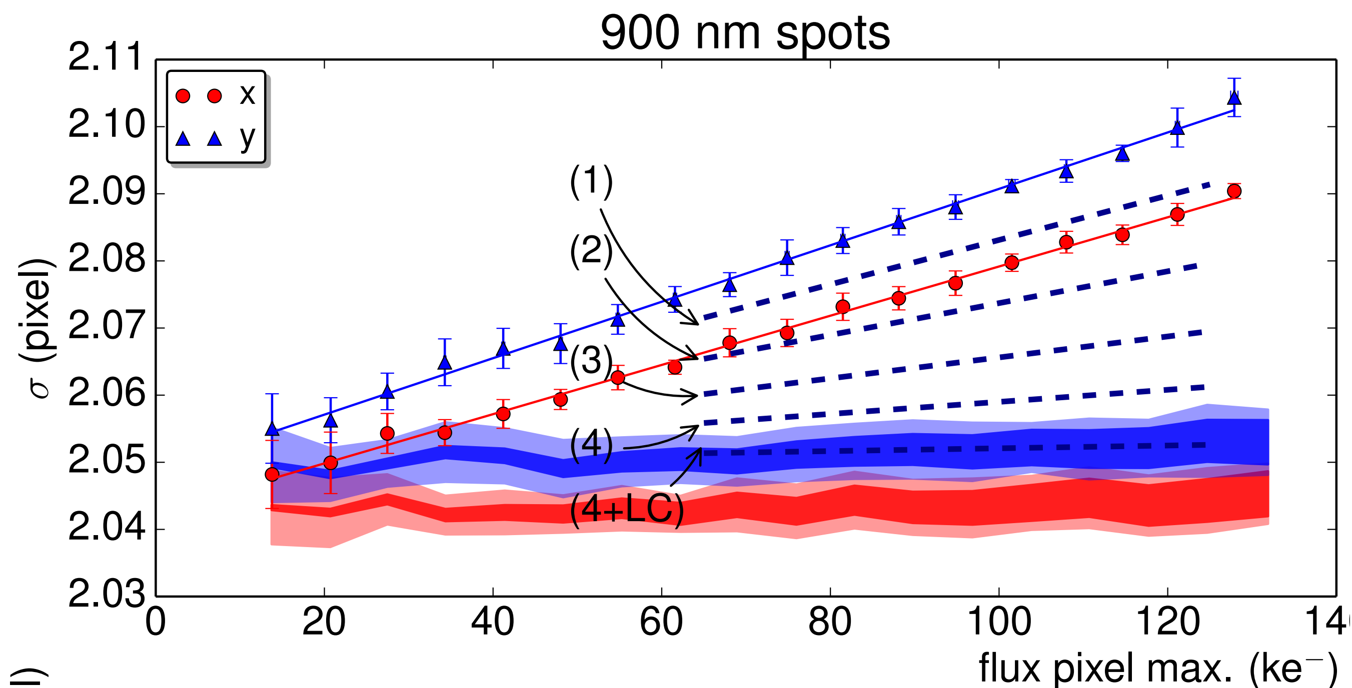
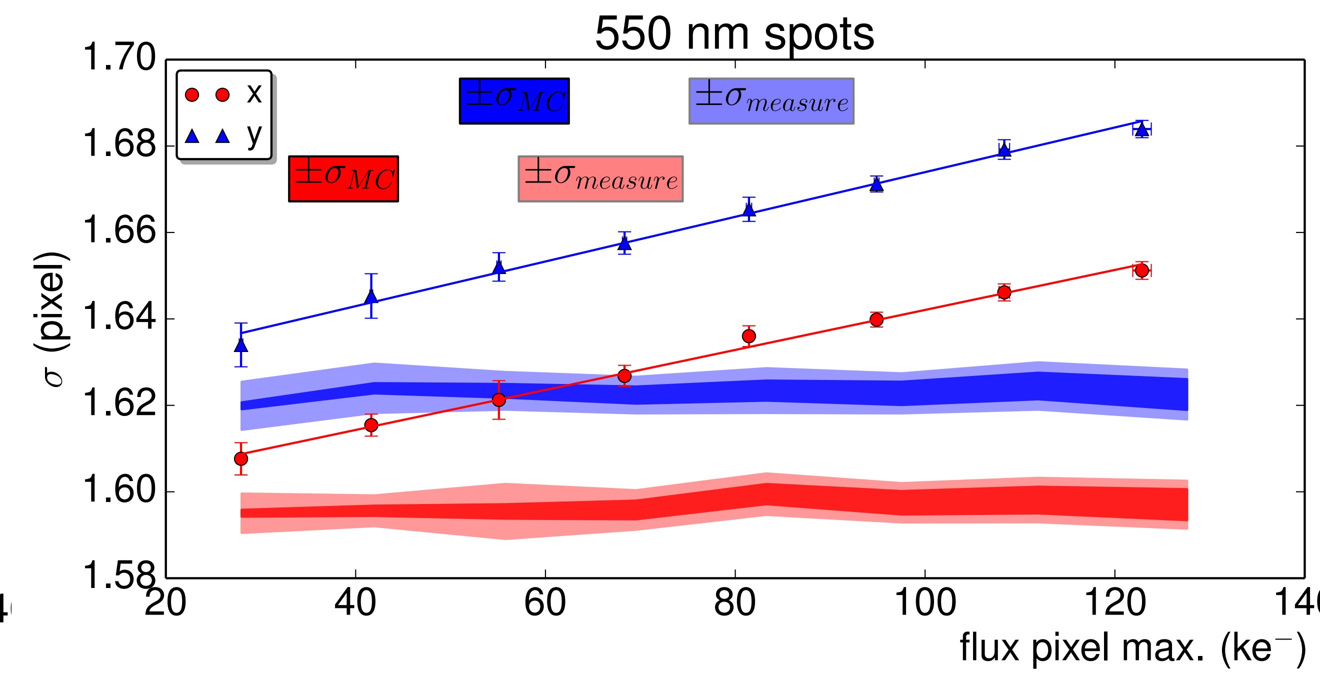
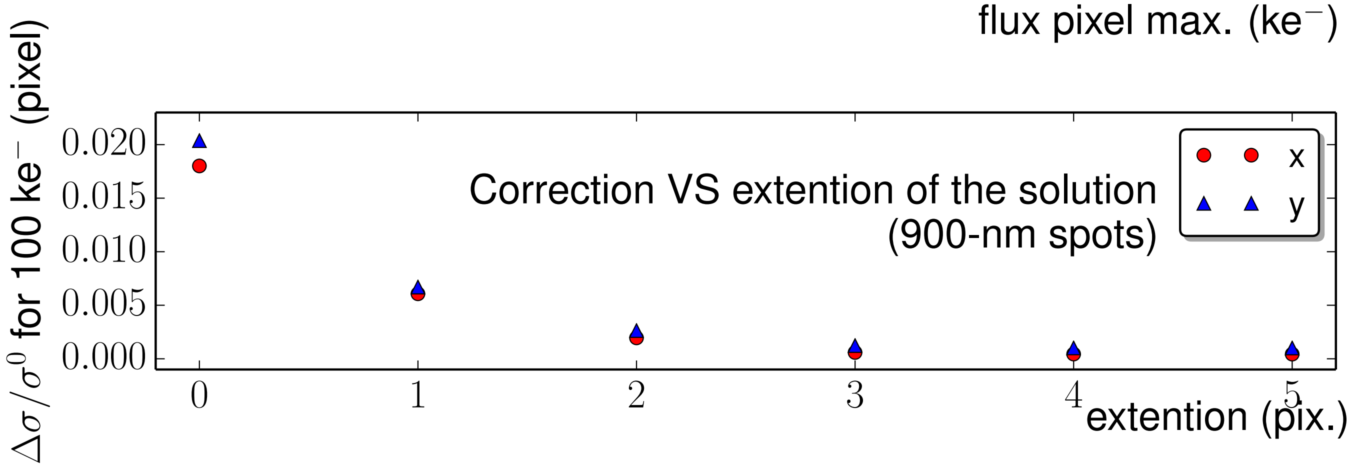
Correction of the «brighter-fatter effect»







	mesurements		corrected
Fit parameters	Slopes $\pm\sigma_{meas.}$ $[10^{-4}\text{pix/ke}]$	origin [pix]	Slopes $\pm(\sigma_{meas.} \oplus \sigma_{model.})$ $[10^{-4}\text{pix/ke}]$
X - 550nm	4.61 ± 0.17	1.594	0.21 ± 0.23
Y - 550nm	5.06 ± 0.18	1.622	0.04 ± 0.20
X - 900nm	3.80 ± 0.05	2.042	0.15 ± 0.17
Y - 900nm	4.25 ± 0.06	2.048	0.22 ± 0.13

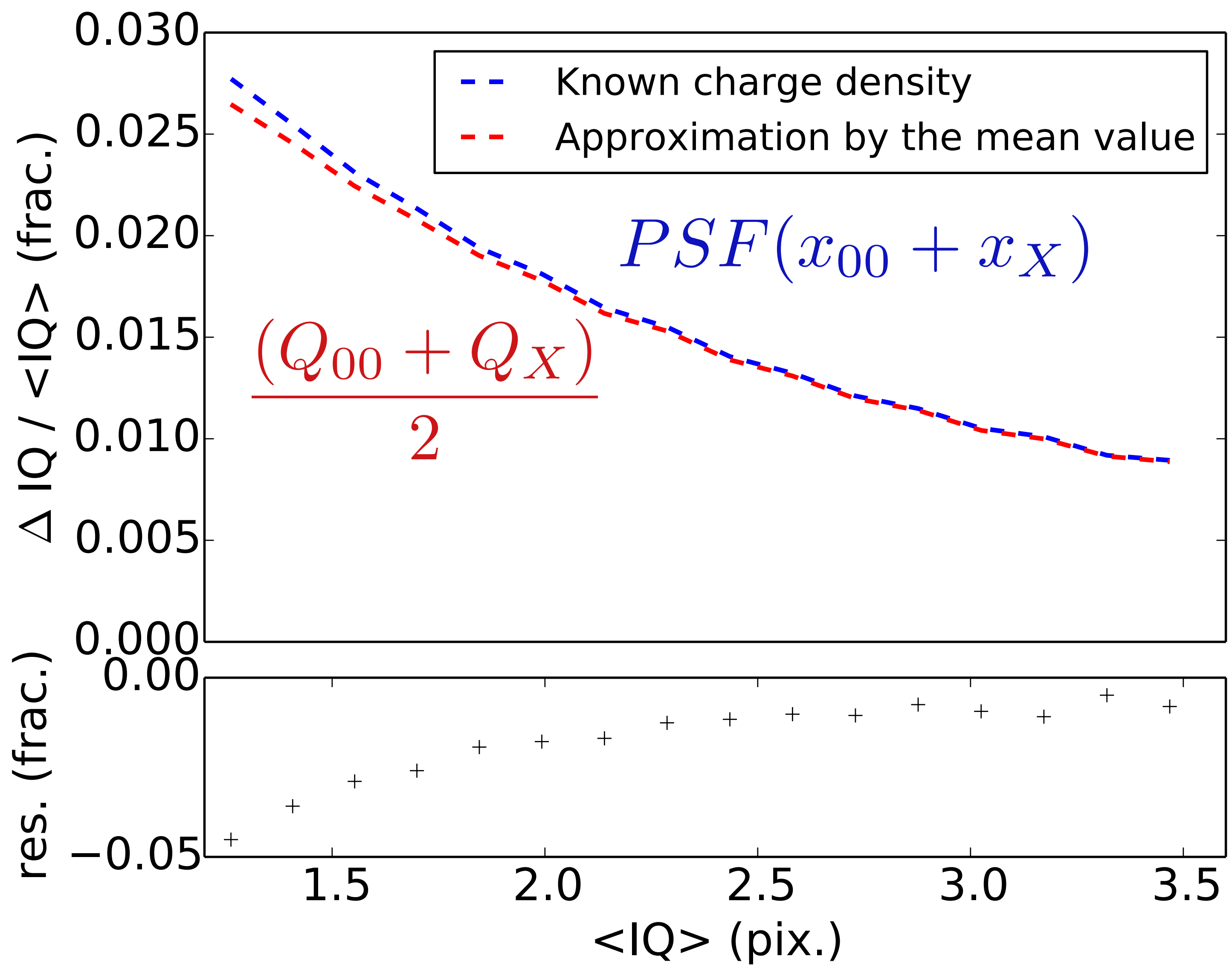
Parameters of linear fits on the E2V CCD-250 data.

The correction has a :
≃5% relative precision
≃5% positive residual

The charge density approximation and image sampling

Evaluating the impact of
the charge density approximation
on the correction :

$$Q'_{0,0} = Q_{0,0} + \frac{1}{4}\sum_X \sum_{i,j} a^X_{i,j} Q_{i,j} (Q_{0,0} + Q_X)$$



=> The approximation underestimate the effect by about 4% @ IQ =1.6 pix 2% @ IQ =2 pix.

Principal steps of our method to remove point source broadening

Measurement of pixel correlation coefficients on flatfields

An assumption on boundaries displacement

pixel effective size model

Projection of the electrostatic effect

solving the model

Charge density approximation

redistribution of the charges